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THERMAL INSTABILITY OF PLANE POISEUILLE FLOW IN THE THERMAL ENTRANCE REGION

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Abstract—The onset of thermal convection in plane Poiseuille flow is investigated theoretically. New stability equations are derived by using the propagation theory considering the variations of disturbance amplitudes in the main flow direction. In the thermal entrance region an analytical procedure to predict the critical conditions for extremely small Prandtl-number fluids is described, based on the local similarity. For $x_c \leq 0.01$ the critical Rayleigh numbers are well represented in the whole domain of the Prandtl number by

$$Ra_c = 200(1 + 0.123Pr^{-1})x_c^{-1}$$

under the conventional boundary layer theory. It is of much interest that the time-independent, three dimensional disturbances become more stable with a decrease in the Prandtl number.

INTRODUCTION

When a horizontal fluid layer is heated from below, buoyancy-driven convection occurs at high heating rates. In order to make an accurate heat transfer prediction it is necessary to examine the stability of a fluid layer. Furthermore the onset conditions of natural convection may be deeply interrelated with turbulent thermal convection, as was indicated by Howard[1]. The occurrence of longitudinal vortex rolls in plane Poiseuille flow due to buoyant force has been observed by several investigators[2-8].

The instability problem in the thermal entrance region for plane Poiseuille flow of a Newtonian fluid layer heated isothermally from below was analysed first by Hwang and Cheng[9]. In their analysis the disturbance amplitudes were assumed to be independent of the main flow direction. Their predictions on the critical Rayleigh number marking the onset of thermal convection were two orders of magnitude lower than the existing experimental data of water and air [3-5,8]. Yeo and Choi[10] analysed this problem by assuming the bottling effect of temperature disturbances confined within the thermal boundary layer. For large Prandtl numbers their analytical predictions were close to experimental data.

Very recently Kim and Choi[11] reformulated the stability equations by using the propagation theory introduced by Choi et al.[12], wherein the variations of disturbances in the main flow direction are considered. The two predictions of Yeo and Choi[10] and Kim and Choi[11] for large Prandtl number fluids were almost the same. Ahn and Choi[8] complemented the results of Kim and Choi[11] by producing the critical conditions for cases of extremely small Prandtl numbers. They neglected the disturbance of a velocity component in the main flow direction. Their results show that the critical Rayleigh number increases with a decrease in the Prandtl number, on the contrary to those of Hwang and Cheng[9].

The purpose of this present study is to extend the work of Ahn and Choi[8] by generating the new stability equations implying Prandtl's boundary layer concept and producing the related critical conditions. The analytical illustration will be limitted to the systems of extremely small Prandtl numbers.

THEORETICAL ANALYSIS

1. Base Temperature

The system considered here is the thermal entrance region of a Newtonian fluid confined between two horizontal infinite plates as shown in Fig. 1. The fluid temperature is uniform at T_1 for $X<0$ and there is a step change in the bottom temperature to a higher value T_2 at the position $X = 0$. The fluid properties are assumed constant and the viscous dissipation of energy is neglected. The velocity field is fully developed in the form of plane Poiseuille flow. We take the layer depth d as the unit of length, the temperature difference (T_2-T_1) as the unit of base temperature and the mean velocity U_m as the unit of base velocity. Then the